Global Positioning System (GPS):

The Global Positioning System (GPS), originally Navstar GPS, is a satellite-based radio navigation system owned by the United States government and operated by the United States Air Force. It is a global navigation satellite system that provides geo location and time information to a GPS receiver anywhere on or near the Earth where there is an unobstructed line of sight to four or more GPS satellites. Obstacles such as mountains and buildings block the relatively weak GPS signals.

The GPS does not require the user to transmit any data, and it operates independently of any telephonic or internet reception, though these technologies can enhance the usefulness of the GPS positioning information. The GPS provides critical positioning capabilities to military, civil, and commercial users around the world. The United States government created the system, maintains it, and makes it freely accessible to anyone with a GPS receiver.

Now Google maps provide location of person or a place in a mobile phone or computer with the help of internet.

Kepler's laws of planetary motion:

1. *The orbit of a planet is an ellipse with the Sun at one of the two foci.*
2. *A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.*
3. *The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.*

Newton's law of universal gravitation:

Newton's law of universal gravitation states that every particle attracts every other particle in the universe with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{or} \quad F = G\frac{m_1 m_2}{r^2}$$

The constant G is called gravitational constant. Its vale is 6.67×10^{-11} N m^2 kg^{-2}. Since constant is small this force is appreciable in the case of massive bodies like stars, sun, planets etc..

Intensity of gravitational field:

The action at a distance due to gravitation occurs through gravitational field. Every mass is a source of gravitational field. This field acts a force on any mass kept in the field. ***The intensity of gravitational field at a point due to a massive body or particle is defined as the force acting on unit mass placed at that point***. Clearly gravitational field is a vector field. From Newton's law of gravitation we can see that intensity of gravitational field due to a point mass at a point is

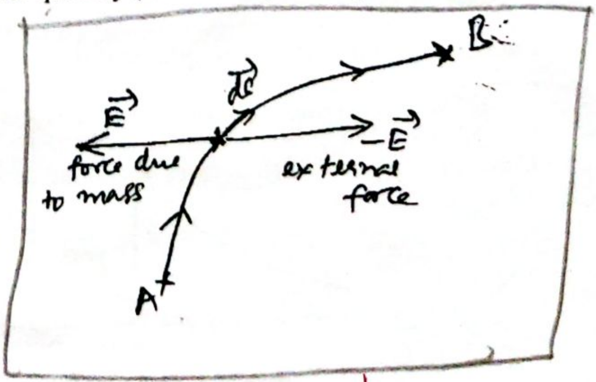
$$\vec{E} = -\frac{Gm}{r^2}\hat{r}$$

where $\hat{r}$ is unit vector in the direction of position vector $\vec{r}$ of the point from the point mass m.

Gravitational potential:

Determination of intensity due to mass distributions directly from Newton's law of gravitation is very complicated. As in the case of electricity here also a scalar quantity (Scalar field) called potential can be defined at every point. Potential can be determined without much difficulty since it is a scalar. From potential field can be calculated. The defining equation of potential is

$$V_B - V_A = -\int_A^B \vec{E}.\vec{dl} \qquad (1)$$



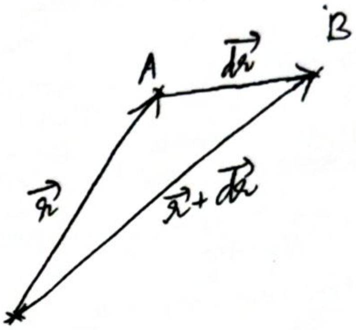
where $V_B - V_A$ is the potential difference between two points A and B and line integral of $-\vec{E}$ (external force) is calculated over any path connecting A and B. Taking point A at infinity we may assume $V_\infty = 0$ and taking B as some point P the potential at P can be written as

$$V(r) = -\int_{\infty}^{P} \vec{E}.\overrightarrow{dl} \qquad (2)$$

Hence *potential due to a mass distribution at any point is the work done by an external force in shifting unit mass from infinity to that point.*

Calculation of intensity of field from potential:

In equation (1) take A and B to be two very close points whose position vectors are $\vec{r}$ and $\vec{r} + \overrightarrow{dr}$ and let potentials are V and $V + dV$ respectively. Now equation (1) becomes



$$(V + dV) - V = -\vec{E}.\overrightarrow{dr}$$

$$dV = -\vec{E}.\overrightarrow{dr}$$

$$\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz = -\vec{E}.\overrightarrow{dr}$$

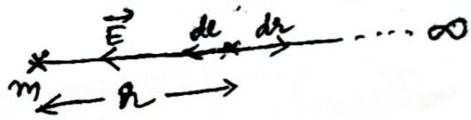
$$\vec{\nabla}V.\overrightarrow{dr} = -\vec{E}.\overrightarrow{dr}$$

Hence we see that
$$\vec{E} = -\vec{\nabla}V$$

This is the relation between potential and field.

Potential due to a point mass:

Let unit mass is brought from ∞ to to the point mass m along the line joining masses. Potential at a point P which is at r away from mass is



$$V(r) = -\int_{\infty}^{P} \vec{E}.\overrightarrow{dl}$$

$$= -\int_{\infty}^{r} Edl\cos 0 = \int_{\infty}^{r} Edr = \int_{\infty}^{r} \frac{Gm}{r^2} dr = -Gm\left(\frac{1}{r} - \frac{1}{\infty}\right)$$

$$= -\frac{Gm}{r}$$

Let us calculate intensity from potential using the equation $\vec{E} = -\vec{\nabla}V$. We have

$$\vec{E} = -\vec{\nabla}\left(-\frac{Gm}{r}\right)$$

Using vector identity $\vec{\nabla}f(r) = \frac{df}{dr}\hat{r}$ we get

$$\vec{E} = Gm\frac{d}{dr}\left(\frac{1}{r}\right)\hat{r} = -\frac{Gm}{r^2}\hat{r}$$

which is the expected result.

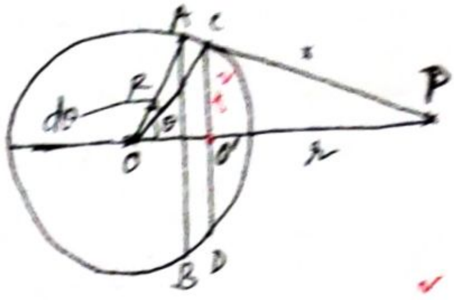
Potential and intensity due to spherical shell of mass:

1) *At a point outside or on the surface of shell,* $r \geq R$

Consider a spherical shell of mass M and radius R. Its mass per unit area σ is given by

$$\sigma = \frac{M}{4\pi R^2} \qquad (1)$$

Cut the shell along two planes AB and CD as shown in figure. The cut portion is a thin ring and all point masses on the ring are at a distance x away from the point P at which the potential is to be calculated. Area of the ring is $2\pi\ R\sin\theta\ Rd\theta$ and mass is $2\pi\ R\sin\theta\ Rd\theta\ \sigma$. The potential due to it is



$$dV = -\frac{G2\pi\ R\sin\theta\ Rd\theta\ \sigma}{x}$$

From ΔOCP $\quad x^2 = r^2 + R^2 - 2rR\cos\theta$

Taking differential $\quad 2x\,dx = 0 + 0 + 2rR\sin\theta\ d\theta$

Hence $\quad \sin\theta\ d\theta = \frac{x\,dx}{rR}$

Therefore $\quad dV = -\frac{G2\pi R^2\sigma}{x}\frac{x\,dx}{rR}$

$O'P = (r - R\cos\theta)$

$x^2 = (O'P)^2 + (O'C)^2$

$= (r - R\cos\theta)^2 + (R\sin\theta)^2$

$= r^2 + R^2\cos^2\theta - 2rR\cos\theta + R^2\sin^2\theta$

$= r^2 + R^2 - 2rR\cos\theta$

Total potential due to entire shell is

$$V = -\frac{G2\pi R^2\sigma}{rR}\int_{r-R}^{r+R} dx$$

$$= -\frac{G2\pi R^2\sigma}{rR}\left(r + R - (r - R)\right)$$

$$= -\frac{G4\pi R^2\sigma}{r}$$

Substituting for σ using (1) we get

$$\boxed{V = -\frac{GM}{r}}$$

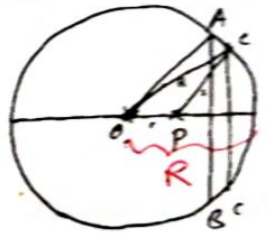
Intensity is $\quad \vec{E} = -\vec{\nabla}V = -\vec{\nabla}\left(-\frac{Gm}{r}\right)$

Using the formula $\vec{\nabla}f(r) = \frac{df}{dr}\hat{r}$ we get

$$\vec{E} = Gm\frac{d}{dr}\left(\frac{1}{r}\right)\hat{r} = -\frac{Gm}{r^2}\hat{r} \qquad \boxed{\vec{E} = -\frac{Gm}{r^2}\hat{r}}$$

Potential and intensity are the same as that of point mass kept at the center of the shell.

2) *At a point inside the shell,* $r < R$



In this calculation only limits of integration change. That is

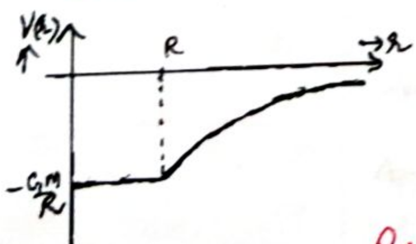
$$V = -\frac{G2\pi R^2\sigma}{rR}\int_{R-r}^{R+r} dx = -\frac{G2\pi R^2\sigma}{rR}\left(R + r - (R - r)\right)$$

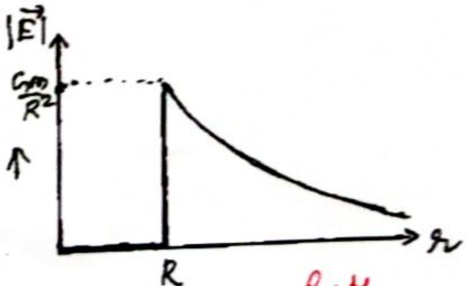
$$\boxed{V = -\frac{GM}{R}}$$

M → constant
R → constant

V = constant

So potential inside shell is constant and is equal to potential at the surface of shell. The intensity inside shell is $\boxed{\vec{E} = -\vec{\nabla}V = 0.}$





$$V = -\frac{G\,2\pi R^2}{rR}\sigma\int_{R-r}^{R+r} dx = -\frac{G2\pi R}{r R}\frac{M}{4\pi R^2}\int_{R-r}^{R+r} dx = -\frac{GM}{2rR}(R+r-R+r)$$

$$= -\frac{GM}{2rR}2r = -\frac{GM}{R}$$

Potential and intensity due to solid sphere of mass:

1) *At a point outside or on the surface of sphere, $r \geq R$*

In this case the sphere may be thought of made up of large number of concentric shells of masses $m_1, m_2, ...$ Since the point is outside or on the surface of each shell we can use the formula of shell in each case. Adding all these we get

$$V = \left(-\frac{Gm_1}{r}\right) + \left(-\frac{Gm_2}{r}\right) + \cdots$$

$$= -\frac{G}{r}(m_1 + m_2 + \cdots)$$

$$\boxed{V = -\frac{GM}{r}}$$

Intensity is $\vec{E} = -\vec{\nabla}V = -\vec{\nabla}\left(-\frac{Gm}{r}\right) = Gm\frac{d}{dr}\left(\frac{1}{r}\right)\hat{r} = -\frac{Gm}{r^2}\hat{r}$

$\therefore \boxed{\vec{E} = -\frac{Gm}{r^2}\hat{r}}$

2) *At a point inside the sphere, $r < R$*

Let the point P is at distance r away from center. The potential can be written as sum of two parts as $V = V_1 + V_2$, where

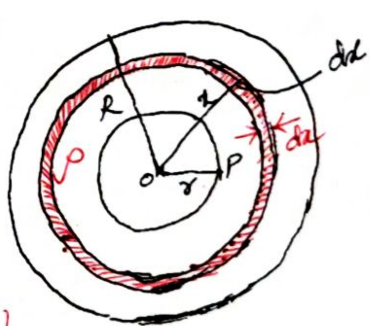
$$V_1 = \textit{potential due to solid sphere of radius } r$$
$$V_2 = \textit{potential due to remaining part of sphere}$$

Since V_1 potential due to solid sphere of radius of radius r on its surface, it is given by

$$V_1 = -\frac{G \times mass}{r} = -\frac{G\left(\frac{4}{3}\pi r^3 \rho\right)}{r} = -\frac{G4\pi\rho r^2}{3}$$

where ρ is the mass per unit volume : $\rho = \frac{M}{\frac{4}{3}\pi R^3}$. To find V_2 imagine a thin shell of radius x and thickness dx as shown in figure. The point P is inside shell and hence potential is equal to potential on the surface given by

$$dV_2 = -\frac{G \times mass}{x} = -\frac{G(4\pi x^2 dx \rho)}{x} = -G4\pi\rho x\, dx$$



Hence V_2 is

$$V_2 = -G4\pi\rho \int_r^R x dx = -\frac{G4\pi\rho}{2}(R^2 - r^2)$$

Hence total potential is

$$V = -G4\pi\rho\frac{r^2}{3} - G4\pi\rho\frac{(R^2 - r^2)}{2}$$

$$= -G4\pi\rho\frac{2r^2 + 3(R^2 - r^2)}{6} = -G4\pi\rho\frac{3R^2 - r^2}{6}$$

Substituting for ρ

$$V = -\frac{G4\pi M}{\frac{4}{3}\pi R^3}\frac{3R^2 - r^2}{6}$$

$\rho = \frac{M}{\left(\frac{4}{3}\pi R^3\right)}$

volume $dV = 4\pi x^2 dx$

Mass of shell $= (4\pi x^2 dx)\rho$

$= \frac{4\pi x^2 dx M}{\frac{4}{3}\pi R^3}$

$= \frac{3Mx^2 dx}{R^3}$

$\therefore dV_2 = -\frac{G\left(\frac{3Mx^2dx}{R^3}\right)}{x}$

$\Rightarrow dV_2 = -\frac{3GMxdx}{R^3}$

$V_2 = -\frac{3GM}{R^3}\int_r^R x dx$

$= -\frac{3GM}{R^3}\left[\frac{x^2}{2}\right]_r^R$

$= -\frac{3GM}{2R^3}(R^2 - r^2)$

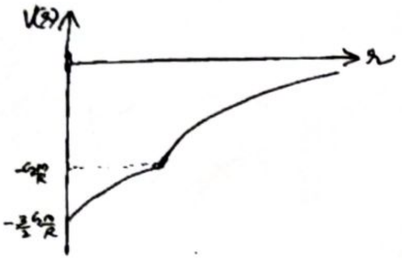
$$\boxed{V = -GM\frac{3R^2-r^2}{2R^3}}$$

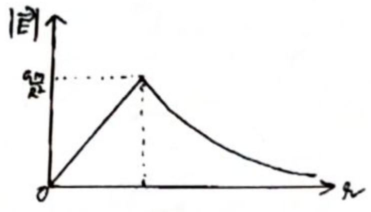
Intensity is given by

$$\vec{E} = -\vec{\nabla}V = -\vec{\nabla}\left(-GM\frac{3R^2-r^2}{2R^3}\right)\hat{r} = \frac{GM}{2R^3}\frac{d}{dr}(3R^2-r^2)\,\hat{r}$$

$$\boxed{\vec{E} = -\frac{GM}{R^3}r\,\hat{r}}$$

Magnitude of intensity is directly proportional to r.





$$V_2 = -\frac{G4\pi r^2\rho}{3} - \frac{3GM}{2R^3}(R^2-r^2)$$

$$= -\frac{G4\pi r^2}{3}\frac{3M}{4\pi R^3} - \frac{3GMR^2}{2R^3} + \frac{3GMr^2}{2R^3}$$

$$= -\frac{GMr^2}{R^3} - \frac{3GMR^2}{2R^3} + \frac{3GMr^2}{2R^3}$$

$$= \frac{GMr^2}{R^3}\Big[$$

$$V_1 = -\left(\frac{G4\pi r^2}{3}\right)\rho$$

$$= -\frac{G4\pi r^2}{3}\frac{3M}{4\pi R^3}$$

$$V_1 = -\frac{3GM}{3R^3}r^2$$

$$V_2 = -\frac{3GM}{3R^3}r^2 - \frac{3GM}{2R^3}(R^2-r^2)$$

$$= -\frac{3GM}{R^3}\left[\frac{r^2}{3} + \frac{R^2}{2} - \frac{r^2}{2}\right]$$

$$= -\frac{3GM}{R^3}\left[\frac{2r^2+3R^2-3r^2}{6}\right]$$

$$= -\frac{3GM}{R^3}\left(\frac{3R^2-r^2}{6}\right)$$

$$V = -GM\frac{(3R^2-r^2)}{2R^3}$$