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Notes on Astrophysics (Elective Paper : Radio Interferometry)

B.Sc 3rd Year (Semester V)

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Part I

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Radio Galaxy

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Extragalactic radio sources are objects that emit a **continuum of radio wavelengths** and were divided in the 1950s into two classes depending on whether they present spatially extended or essentially "starlike" images. Radio galaxies belong to the former class (extended), and quasars (quasi-stellar radio sources) to the latter (star like).

Radio galaxies are types of galaxies which emit more amount of light at radio wavelengths than at the visible wavelengths. These galaxies are also called as radio-luminous or radio-loud galaxies. They have a very high luminosity at radio wavelengths ranging from 10 MHz and 100 GHz. This radio emission is due to the synchroton process which happens inside them. Most common structure of these galaxies are called as lobes which are double often symmetrical structures whereas another variety called as plumus are more elongated.

The most powerful extragalactic sources of radio waves are double-lobed sources (or "dumbbells") in which two large regions of radio emission are situated in a line on diametrically opposite sides of an optical galaxy. The radio waves coming from double-lobed sources are undoubtedly **synchrotron radiation**, produced when relativistic electrons (those traveling at nearly the speed of light : relativistic speed) emit a quasi-continuous spectrum as they gyrate wildly in magnetic fields.

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Synchrotron Radiation :

Synchrotron radiation is electromagnetic energy emitted by charged particles (e.g., electrons and ions) that are moving at speeds close to that of light when their paths are altered, as by a magnetic field.

The synchrotron radiation, the emission of very relativistic and ultrarelativistic electrons gyrating in a magnetic field, is the process which dominates much of high energy astrophysics. It was originally observed in early betatron experiments in which electrons were first accelerated to ultrarelativistic energies. This process is responsible for the radio emission from the Galaxy, from supernova remnants and extragalactic radio sources. It is also responsible for the non-thermal optical and X-ray emission observed in the Crab Nebula and possibly for the optical and X-ray continuum emission of quasars.

Many kinds of astronomical objects have been found to emit synchrotron radiation as well. High-energy electrons spiraling through the lines of force of the magnetic field around the planet Jupiter, for example, give off synchrotron radiation at radio wavelengths. Synchrotron radiation at such wavelengths and at those of visible and ultraviolet light is generated by electrons moving in the magnetic field associated with the supernova remnant known as the Crab Nebula. Radio emissions of the synchrotron variety also have been detected from other supernova remnants in the Milky Way Galaxy and from extragalactic objects called quasars.

Synchrotron radiation characteristically is highly polarized and continuous. Its intensity and frequency are directly related to the strength of the magnetic field and the energy of the charged particles affected by the field. Accordingly, stronger the magnetic field and higher the energy of the particles, the greater the intensity and frequency of the emitted radiation. Synchrotron radiation is not dependent on the temperature of a given astronomical source; a relatively cool object can release substantial amounts of electromagnetic energy in this form. Synchrotron radiation is thus often termed nonthermal radiation.

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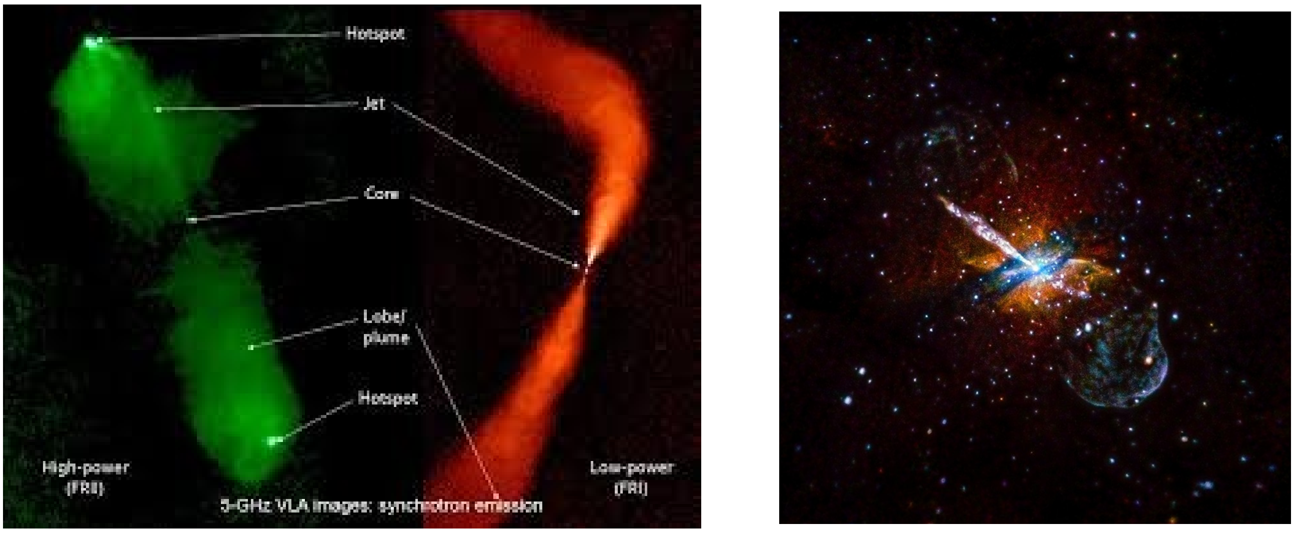

Fig : Images of Radio Galaxy

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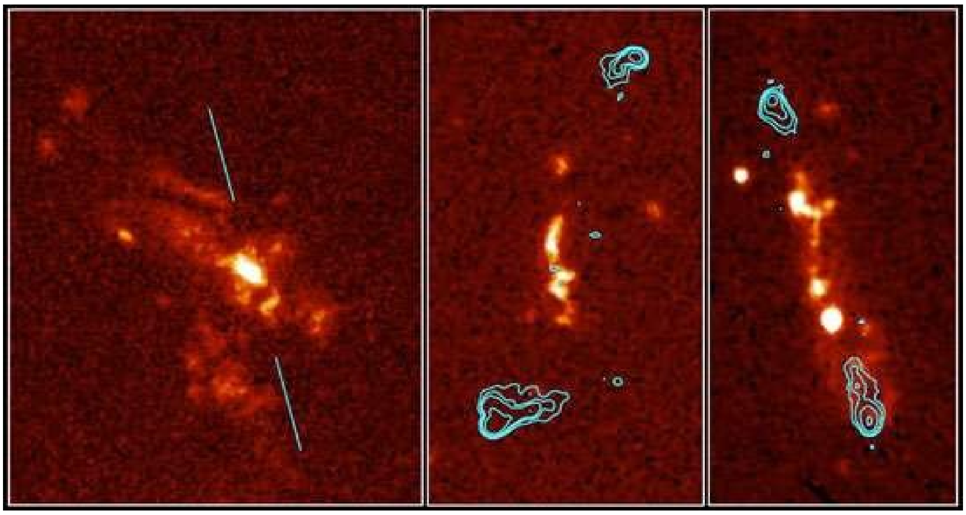
Radio galaxies are powered by the galaxy’s **supermassive black hole (SMBH)**. When a supermassive black hole eats nearby gas, dust and stars, it is known as an **active galactic nucleus (AGN)**. These active black holes have superheated material swirling around them in a circular disk, known as an accretion disk. The material in the accretion disk is swirling around the black hole in a tight circle, which means all the charged particles in the disk are being accelerated. When you accelerate charges, they give off radio waves along their direction of motion, known as synchrotron radiation. If you look at an AGN from a bit of an angle, you would see intense visible light from the superheated accretion disk, and depending on your angle you might (or might not) also see intense radio waves. If that’s the case, then it is a radio galaxy.



One of the more famous radio galaxies is **Centaurus A**, which is one of the closest radio galaxies. Since its distance is only 10-15 million light years, we can actually get good observations. We know, for example, that its supermassive black hole is about 50 million solar masses, and that it is likely active due to a collision with a smaller galaxy.

Radio galaxies are also related to **quasars (quasi-stellar radio source)** as they are bright sources of radio energy that appeared point-like and thus star-like. We now know that quasars are also powered by active galactic nuclei, and ***radio loud quasars are distant radio galaxies***.

The nucleus of a so-called "active" galaxy contains a massive black hole that is vigorously accreting material. As a result, the nucleus often ejects bipolar jets of rapidly moving charged particles that radiate brightly at many wavelengths, in particular radio wavelengths. Active galaxies display a range of dramatically different properties, and the ones that are bright in the radio can beam as much as one trillion solar luminosities of radiation into space at those wavelengths.

The intense emission arises from the hot environment of the black hole because electrons, moving at close to the speed of light in an environment of strong magnetic fields, radiate in the radio wavelength. The directed particle jets eventually collide with the ambient medium and convert much of their bulk energy of motion into shocks. The points of termination in the jet flow are seen as very hot spots, bright and compact structures. The hotspots can reverse the flow of the jets back towards the black hole, and thereby generate additional turbulence and random motions. The characteristic temperature of a hot spot reveals the nature of the physical processes at work.

Properties of Radio Galaxies :

- Dominated by synchroton emission (from accelerated particles), referred also as non-thermal emission.
- The synchrotron emission is affected by:
  - self-synchrotron absorption (low-frequency)
  - free-free absorption (low frequency)
  - inverse-Compton (re-emitted at X-ray wavelengths)

- The nature of the emission of the jets makes it highly anisotropic: possible to use this property to extract information about the inclination.
- The IR emission from the dust absorbing the UV- optical light from the AGN and re-emitting in mid-IR is also a proxy for inclination (fraction of very hot central dust seen by the observer)

Radio Telescopes in India

1. Giant Metrewave Radio Telescope (GMRT), located near Pune.

Thirty 45 m wire dishes; largest telescope at meter wavelength. Operated by the National Centre for Radio Astrophysics.

2. Ooty Radio Telescope, Tamilnadu.

Approximately 530 m long and 30 m wide.

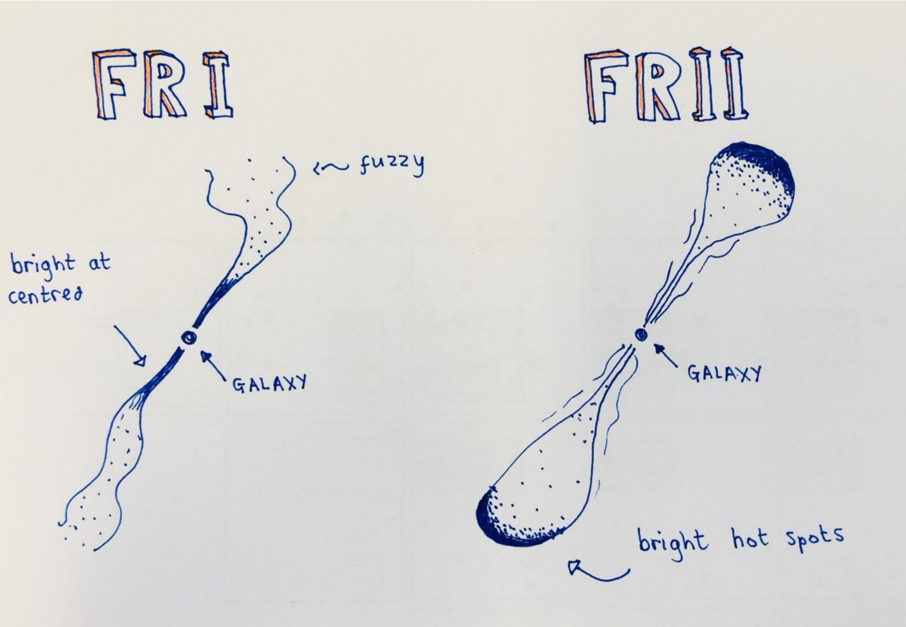
3. Gauribidanur Radio Observatory, Karnataka, India

Operated by (Indian Institute of Astrophysics). It is a Radioheliograph.

Classification of Radio Galaxies (Fanaroff–Riley classification)

The **Fanaroff–Riley classification** is a scheme created by B. L. Fanaroff and J. M. Railey in 1974, which is used to distinguish radio galaxies with active galactic nuclie based on their radio luminosity or brightness of their radio emission in relation to their hosting environment.

- ***Class I*** (abbreviated **FR-I**) are sources whose luminosity decreases as the distance from the central galaxy or quasar host increase.
- ***Class II* (FR-II)** sources exhibit increasing luminosity in the lobes. These sources are called also *edge-brightened*.



This distinction is important because it presents a direct link between the galaxy's luminosity and the way in which energy is transported from the central region and converted to radio emission in the outer parts.

Fanaroff and Riley studied a sample of 57 radio galaxies and quasars that were clearly resolved at 1.4 GHz or 5 Ghz into two classes using a ratio, $\boldsymbol{R}_{\mathbf{FR}}$, of the distance between the regions of highest surface brightness on opposite sides of the central galaxy or quasar to the total extent of the source up to the lowest brightness contour in the map. Sources with $\boldsymbol{R}_{\mathbf{FR}} \mathbf{< 0.5}$ were placed in **Class I** and sources with $\boldsymbol{R}_{\mathbf{FR}} \mathbf{> 0.5}$ in **Class II**. Various properties of sources in the two classes are different, which is indicative of a direct link between luminosity and the way in which energy is transported from the central region and converted to radio emission in the outer parts.

Difference between FR I and FR II

Fanaroff-Riley Class I (**FR-I**) sources have their low brightness regions further from the central galaxy or quasar than their high brightness regions. The sources become fainter towards the outer extremities of the lobes. Here, the spectra are steepest, indicating that the radiating particles have aged the most. Jets are detected in 80% of FR-I galaxies, which also tend to be bright. They are usually associated with large galaxies that have a flatter light distribution than an average elliptical galaxy and are often located in rich clusters of galaxies.

Fanaroff-Riley Class II (FR-II) is made up of luminous radio sources with hotspots in their lobes at distances from the center such that $R_{\mathrm{FR}} > 0.5$. These sources are said to be **edge-darkened. I**n keeping with the overall high luminosity of this type of source, the cores and jets in them are also brighter than those in FR-I galaxies in absolute terms, but relative to the lobes these features are much fainter in FR-II galaxies. Jets are detected in less than 10% of luminous radio galaxies, but in nearly all quasars. FR-II sources are generally associated with galaxies that appear normal, except that they have nuclear and extended emission line regions. The galaxies are giant ellipticals, but not first-ranked cluster galaxies.

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Part II

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Quasars

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A **quasar** (also known as a **quasi-stellar object** abbreviated **QSO**) is an extremely luminous active galactic nucleus (AGN), in which a supermassive black hole is surrounded by a gaseous accretion disk. The observed properties of a quasar depend on many factors including the mass of the black hole, the rate of gas accretion, the orientation of the accretion disk relative to the observer, the presence or absence of a jet, and the degree of obscuration by gas and dust within the host galaxy.

Quasars are the most distant and luminous objects in the universe. There are very few energy sources that produce enough energy to power a quasar. The possible source that best fits the observed properties of quasars is a supermassive black hole. At the center of a quasar, the black hole is surrounded by a large, rotating cloud of gas. As the gas falls into the black hole, it is heated up to millions of degrees. The gas emits thermal radiation due to its enormous heat. This thermal radiation spans the spectrum, making the quasar bright in the visible spectrum as well as x-rays. The closest quasar is about 800 million light years away. Therefore, we can conclude that there are no quasars in the universe today and the last quasar disappeared about 800 million years ago.

Physical Properties

- Star like object identified with a radio source.
- variable light;
- large ultraviolet flux of radiation;
- broad emission lines in the spectra with absorption lines in some cases;
- large redshift.

Radio Loud (10 %)	**Radio Quiet (90 %)**
High spin black holes	Low spin black holes
Produce jets, which are the origin of radio emission.	No Jets
Jets powered by spin energy extracted from black hole.	
Also have accretion disks	
	Spectrum produced by the accretion disk (blackbody + nonthermal emission)

Red Shift in Quasars

Redshift, is defined as the displacement of the spectrum of an astronomical object toward longer (red) wavelengths. It is generally attributed to the Doppler Effect, a change in wavelength that results when a given source of waves (e.g., light or radio waves) and an observer are in rapid motion with respect to each other. Edward Powell Hubble reported in 1929 that the distant galaxies were receding from the Milky Way system, in which Earth is located, and that their redshifts increase proportionally with their increasing distance. This generalization became the basis for what is called **Hubble's law**, which correlates the recessional velocity of a galaxy with its distance from Earth. *Quasars exhibit larger redshifts than any of the remotest galaxies previously observed.* The extremely large redshifts of various quasars suggest that they are moving away from Earth at tremendous velocities (i.e., approximately 90 percent the speed of light) and thereby constitute some of the most distant objects in the universe.

Additional Information :

Light behaves like a wave, so light from a luminous object undergoes a Doppler-like shift if the source is moving relative to us. Ever since 1929, when Edwin Hubble discovered that the Universe is expanding, we have known that most other galaxies are moving away from us. Light from these galaxies is shifted to longer (and this means redder) wavelengths - in other words, it is 'red-shifted'. Since light travels at such a great speed relative to everyday phenomena (a million times faster than sound) we do not experience this red shift in our daily lives. The red shift of a distant galaxy or quasar is easily measured by comparing its spectrum with a reference laboratory spectrum. Atomic emission and absorption lines occur at well-known wavelengths. By measuring the location of these lines in astronomical spectra, astronomers can determine the red shift of the receding sources. However, to be accurate, the red shifts observed in distant objects are not exactly due to the Doppler phenomenon, but are rather a result of the expansion of the Universe. Doppler shifts arise from the relative motion of source and observer through space, whereas astronomical redshifts are 'expansion redshifts' due to the expansion of space itself. Two objects can actually be stationary in space and still experience a red shift if the intervening space itself is expanding. A convenient analogy for the expansion of the Universe is a loaf of unbaked raisin bread. The raisins are at rest relative to one another in the dough before it is placed in the oven. As the bread rises, it also expands, making the space between the raisins increase. If the raisins could see, they would observe that all the other raisins were moving away from them although they themselves were stationary within the loaf. Only the dough - their 'Universe' - is expanding.

Properties of Quasars

1. They are found with a number of bright emission lines. By contrast, a normal galaxy has only absorption lines, due to the atmospheres of the member stars. To have emission lines, something must be exciting the atoms.
2. All have large redshifts, ranging from $z = 0.06$ to at least $z = 6.42$. Most quasars have redshifts of 0.3 or more, which implies that they are more than 1000 Mpc (3 billion light years) away.
3. Spectral lines of many other quasars were soon identified to be ordinary lines which had undergone even larger redshifts.
4. Quasars emit very strongly in UV (8×10^{14} to 3×10^{16} Hz). (10–400 nm).
5. They are stellar-looking objects for which the UV brightness compared to the optical brightness is much higher than what we expect in the case of a star.
6. A quasar’s luminosity can be calculated from its apparent brightness and distance using the Inverse-Square Law. A bright quasar is a thousand times more luminous than the entire Milky Way Galaxy.
7. Quasars have a decidedly non-thermal spectrum: they are luminous in the X-rays, ultraviolet, visible, infrared, and radio bands.

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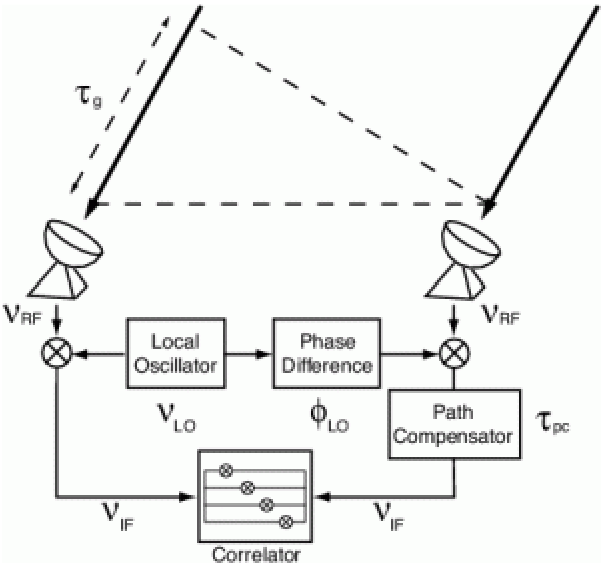
Part III

Radio Interferometry

One of the most powerful techniques of radio astronomy is the use of interferometry to combine the signals of several radio antennas into a single virtual telescope. Through interferometry we can make radio images with resolutions greater than that of the Hubble telescope. To obtain higher spatial resolution for a given wavelength, radio astronomers applied the technique of interferometry. This involves collecting electromagnetic radiation (eg radio waves) using two or more collectors (eg radio antennae) separated by some distance. In interferometry, the resolution then is not determined by the size of individual collectors such as mirrors or radio dishes but rather by the maximum separation of the collecting elements.

At a basic level, interferometry is simply the combining of signals from two different sources. If the two signals are similar they will combine to make a stronger signal, and if they aren't they will tend to cancel out. In astronomy, if two signals are out of sync you can shift them (correlate them) so that they are in sync. When the signal is strongest you know they are lined up.

When two radio antennas are aimed in the same direction, they receive the same basic signal, but the signals are out of sync because it takes a bit longer to reach one antenna than the other. That difference depends on the direction of the antennas and their spacing apart from each other. By correlating the two signals, you can determine location of the signal in the sky very precisely. It's the precision that you need to create a high resolution image.

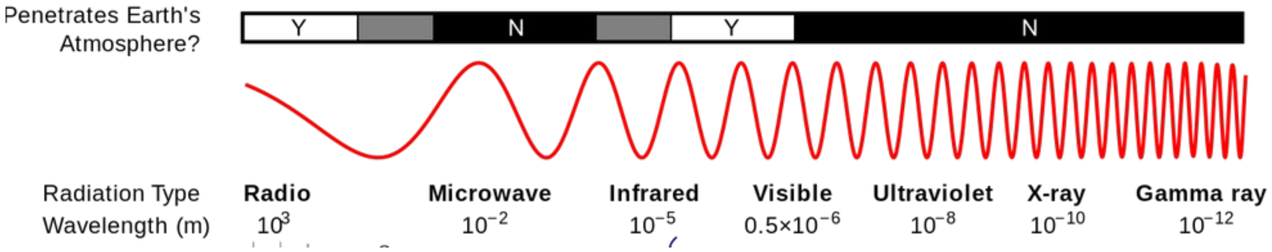


Two antennas only give you one point in the sky, but with dozens of antennas (such as the array at ALMA) you can get lots of points, one for each paring of antennas. But even that only gets you a discrete set of image points. If the Earth were fixed in relation to the sky, then our radio image would look like a pointillist painting.

But the Earth rotates with respect to the sky, so as time goes by the relative positions of the antennas shift with respect to an astronomical signal. As you make observations over time, the gaps between antennas are filled to create a more solid image. This isn't easy to do. It takes lots of observations and lots of computing power to combine the images in the right way. At ALMA, for example, it takes a custom built supercomputer that spends all its time correlating signals.

Basics of Orbiting Telescopes

Space telescopes have the advantage of being above the blurring effects of the Earth's atmosphere. In addition, there are many wavelengths from the electromagnetic spectrum that do not reach Earth because they are absorbed or reflected by the Earth's atmosphere. In fact, it is only the wavelengths of visible light, a small portion of infrared, and a part of the radio wavelengths that reach Earth at all. To observe ultraviolet, x-rays, gamma rays or infrared, astronomers have had to put telescopes outside of Earth's atmosphere -normally in orbit around the Earth.



Infrared Astronomy

Water vapor in the atmosphere absorbs much of the infrared radiation from space so the infrared observatories on Earth are located on high, dry mountains such as Mauna Kea in Hawaii. Even at high altitudes, however, the quality of observations in infrared is limited. The best solution for infrared observing is to put a telescope in orbit above the Earth and use radio to send data back to Earth. The Herschel Space Observatory was launched in May 2009 and the Spitzer Space Telescpoe was launched in August 2003. Herschel's primary mirror is 3.5 meters in diameter and the telescope is designed for infrared wavelengths from 55 to 672 µm. Spitzer's primary mirror is 0.85 meters across and it is designed for wavelengths of 3 to 180 µm. Infrared observatories in space must be kept very cold because otherwise infrared radiation from the telescope itself would interfere with its ability to observe infrared radiation from space. Spitzer exhausted its liquid helium coolant in 2009 and only a few of its instruments are still being used. Herschel's mission came to an end in 2013, when it also ran out of coolant.

Astronomers study the infrared wavelengths to study the early universe and to learn about objects that are too cold to generate visible light including brown dwarf stars and dust clouds.

Visible Wavelengths

Visible wavelengths make it through Earth's atmosphere, but turbulence in the atmosphere causes images of stars to be blurred and spread out by at least 0.5", even at the best observing sites in the world. The Hubble Space Telescope observes from an orbit about 559 km above the Earth at wavelengths from near infrared through the visible range and into the ultraviolet. It has a 2.4 meter primary mirror. It was put into orbit in 1990 and had a major repair in 1993. In May 2009 it was serviced again and should last until its successor, the James Webb Space Telescope (JWST), is launched in 2021. The JWST will be optimized for infrared observation, however, and ground based observatories will be the main source of observations in the visible range when Hubble is no longer able to operate. The Keplar Space Telescope was launched in March, 2009 and was active until November 2018. It had a 1.4 meter diameter primary mirror, and observed in the visible and infrared range of wavelengths. It had a very large field of view and was used to search for Earth sized planets within our galaxy. It detected 2,662 planets during its mission.

X-ray Astronomy

X-ray telescopes make it possible to study objects with temperatures between 10^6 and 10^8 K (between about 1 million and 100 million °C). When atoms in a gas are this hot, they move so fast that when they collide, they emit X-ray photons with wavelengths less than 10 nm. The Earth's atmosphere blocks all X-rays from space, so space telescopes must be used to observe in these wavelengths. X-rays have such high energy that the typical reflecting telescope design used for radio, infrared and optical telescopes cannot be used as the X-rays would just penetrate into the mirror. Instead, mirrors are arranged in specially shaped tubes so that the X-rays coming into the telescope just skim off the surface of the mirror (similar to skipping a stone on the surface of a lake) and onto a detector. Two X-ray telescopes currently in space are the Chandra X-ray Observatory and the XMM-Newton.

Gamma-Ray Astronomy

Gamma-ray telescopes such as the Fermi Gamma Ray Space Telescope do not use mirrors at all and instead have special detectors to measure the energy and direction of the most energetic electromagnetic radiation in the universe, gamma-rays. The Fermi Gamma-ray Space Telescope detects gamma-rays with energies from 10 keV to 300 GeV and has a very large field of view. It sees approximately 20% of the sky at once, and can cover the entire sky every three hours. Studying gamma-rays helps astronomers learn more about many things including active galactic nuclei, blazars, gamma-ray bursts, pulsars and solar flares.

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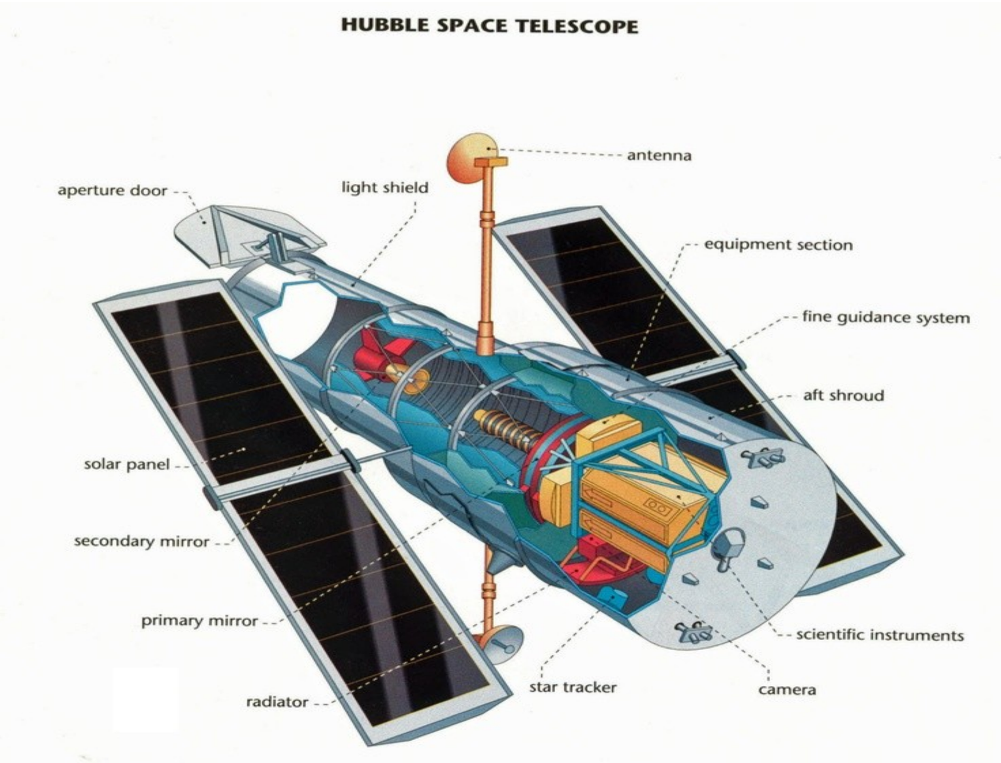
Hubble Space Telescope

NASA's Hubble Space Telescope was the first astronomical observatory to be placed into orbit around Earth with the ability to record images in wavelengths of light spanning from ultraviolet to near-infrared. Launched on April 24, 1990, aboard the Space Shuttle Discovery, Hubble is currently located about 340 miles above Earth's surface where it completes 15 orbits per day. The satellite moves at the speed of about five miles (8 km) per second, fast enough to travel across the United States in about 10 minutes.



The telescope, a basic reflector with a 94.5-inch (2.4-meter) mirror, was packed with instruments that would give astronomers clear views of the universe in visible, infrared and ultraviolet light. Without the Earth's atmosphere blocking its view, Hubble would be able to observe details of astronomical objects that had never been seen before. Light hitting the telescope's main, or primary, mirror is reflected to a smaller, secondary mirror suspended above the primary. The secondary, in turn, reflects the light back through a hole in the primary where it enters Hubble's instruments (cameras and spectrographs) for final focus before it hits their detectors. Hubble's primary mirror is not only exquisitely polished, but at 94.5 inches (2.4 m) in diameter, collects an immense amount of light. Hubble can detect objects that are 10 billion times fainter than the unaided eye can see. High above the blurring effects of Earth's atmosphere, Hubble also gets a much clearer view of the cosmos than do telescopes located on the ground. The space telescope can distinguish astronomical objects with an angular diameter of a mere 0.05 arcsecond This resolution is about 10 times better than the best typically attained by even larger, ground-based telescopes. High resolution enables Hubble to locate such objects as dust disks around stars or the glowing nuclei of extremely distant galaxies. Also because it circles above the atmosphere, Hubble can view astronomical objects across a wider range of the electromagnetic spectrum than ground-based telescopes, which are limited by atmospheric absorption at various wavelengths. This gives astronomers using Hubble a fuller view into the energetic processes that create the radiation seen and measured. Finally, Hubble's observations are predictably consistent. The telescope's seeing conditions do not change from day to day or even orbit to orbit. Astronomers can revisit targets with the expectation that they will be imaged at the same high quality each time. This optical stability is critical for detecting tiny motions or other small variations in celestial objects. Such is not the case for ground-based observatories, where observing conditions vary with weather and directly affect the quality of the images acquired.

Important Discoveries by HST

- Revealed extraordinary details about the process by which Sun-like stars end their lives as planetary nebulae.
- Helped pin down the age for the universe now known to be 13.8 billion years, roughly three times the age of Earth.
- Discovered two moons of Pluto, Nix and Hydra.
- Helped determine the rate at which the universe is expanding.
- Discovered that nearly every major galaxy is anchored by a black hole at the centre.
- Created a 3-D map of dark matter.

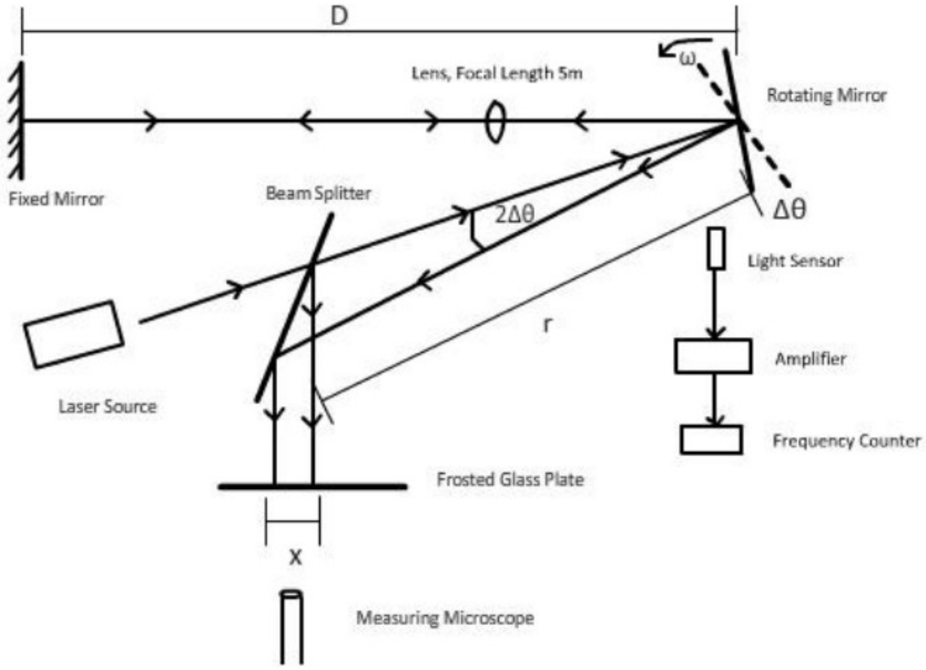
Foucault's Experiment

The goal of the experiment was to experimentally measure the speed of light, c, in a vacuum by using the Foucault method for measuring the speed of light. The underlying principle behind this methods is the simple kinematic relationship between constant velocity, distance and time given below

$$\mathbf{c = D/t} \qquad \text{(i)}$$

Foucault’s method used a light source and rotating mirror together to derive the speed of light. The Foucault method uses the light source to produce a focused beam on the rotating mirror. The light from the rotating mirror is then reflected at an angle to a fixed mirror which is aligned to face perpendicular to the reflected light beam. Therefore the light is reflected directly back to the rotating mirror where it was first reflected. During the time the light had traveled the distance between the two mirrors, the rotating mirror had changed its orientation to the beam of light, thus the returning beam of light will be reflected off at a separate angle. The difference in the angle between the light source to the rotating mirror and the rotating mirror the second reflected beam is related to the time that was required by the light to travel the distance between the fixed and rotating mirrors. Using the relations of the experimental setup, Equation (i) was used to determine the speed of light.

The experimental set up is as shown in the figure. The experiment used a laser to provide a light beam because it creates a focused beam of light to travel between all components. The rotating mirror used was a double sided plane mirror attached to a motor apparatus that allowed a variable control of the motor rotation speed. The rate of rotation was measured using a light sensor in conjunction with a frequency counter.



The lens focused the laser beam directly to a point source on both mirrors making the beams more precisely optically aligned. A beam splitter was used in order to redirect the returning light beam onto a frosted glass plate. The difference between these two points on the frosted glass plate was measured by using the measuring microscope placed in front of the frosted glass screen and recording the value of displacement of the two beams.

Derivation

Here, the distance and time measured were for a beam of light to travel between the rotating and fixed mirrors and back. Therefore, the time to travel the distance can be writen as

$$\mathbf{t = 2D/c} \qquad \text{(ii)}$$

However, during the time period t, the rotating mirror is held at a fixed angular velocity of ω. Because the angular velocity is constant, we have

$$\boldsymbol{\omega_{mirror} = \Delta\theta/t} \qquad \text{(iii)}$$

The angular dependence Δθ is also related to the return of the light beam to the beam splitter and the displacement Δx of the light beam on the frosted glass plate. Therefore for every angular change in the rotating mirror by Δθ, the angle between the returning beam and the incoming beam (deviation angle) should be twice this angle Δθ.

Using the geometry we can write $\boldsymbol{\tan(2\Delta\theta) = \Delta x/r}$

$$\boldsymbol{2\Delta\theta = \Delta x/r}$$

$$\boldsymbol{\Delta\theta = \Delta x/(2r)}$$

$$\boldsymbol{\omega_{mirror} = \Delta x/2rt}$$

$$\boldsymbol{t = \Delta x/2r\,\omega_{mirror}}$$

$$\boldsymbol{2D/c = \Delta x/2r\,\omega_{mirror}}$$

$$\boldsymbol{c = 4rD\,\omega_{mirror}/\Delta x}$$

But, we have $\boldsymbol{\omega_{mirror} = 2\Pi\, f_{mirror}}$

Therefore, $\boldsymbol{c = 8\Pi rD\, f_{mirror}/\Delta x}$

The frequency measured on the frequency counter is twice that of the rotation frequency because the mirror rotated is doubled sided so then the laser beam is reflected into the light sensor twice per each rotation of the mirror.

$$\boldsymbol{F_{mirror} = 1/2\,(f_{measured})}$$

$$\boldsymbol{c = 4\Pi rD\, f_{measured}/\Delta x}$$

Where r is path from the rotating mirror to the beam splitter to the frosted screen.

D is the distance between the fixed mirror and the rotating mirror.

Δx is the displacement of the laser beam across the frosted glass screen