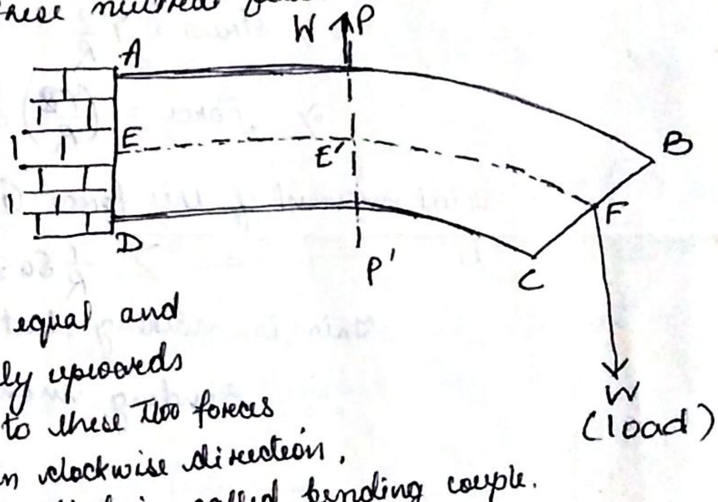
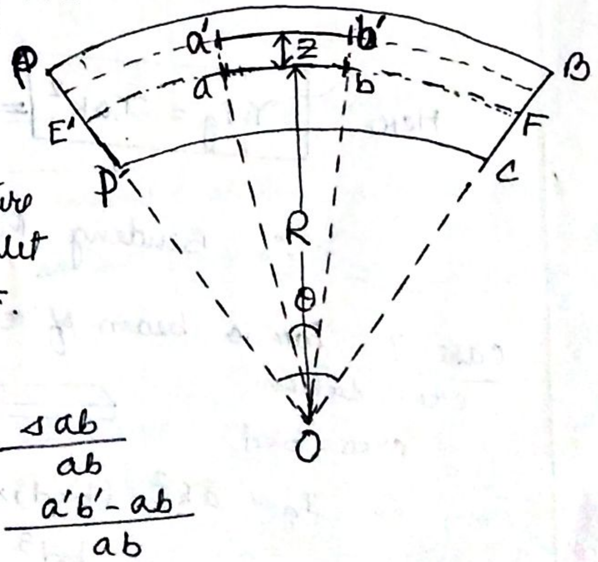
Bending of Beam : **Beam :**
A beam is a rod of uniform cross section whose length is much greater than its other dimensions. They are usually set in horizontal position and are designed to support heavy loads. They are used in buildings to support roofs and in bridges to support load of heavy vehicles.

Neutral filament :
under the action of a couple which tries to bend the a beam, the filaments in the convex side of the beam are extended while those on concave side are shortened. In between these two types of filament, one type of filament is there which neither extends nor shortens. These filaments are called neutral filament. and the plane in the beam containing only these neutral filaments is called neutral plane.



Expression of bending moment of a beam :-
Let us consider ABCD is a beam fixed along AD and loaded at F, with EF being the neutral axis. Let us consider the equilibrium section PBCP' of the beam. A weight W is acting from F in downward direction. So an equal and opposite reactional force W must act vertically upwards along PP'. The couple of force formed due to these two forces tend to rotate the beam or bend the beam in clockwise direction. The couple of force produced due to this load applied is called bending couple. The moment of this couple is called bending moment.



Let us consider a small part of the beam be bent in the form of a circular arc, substending an angle θ at the centre O. Let R is the radius of curvature for the part of the neutral axis E'F and let a'b' be the element at a distance z from E'F.

$\therefore$ $ab = R\theta$; $a'b' = (R+z)\theta$

$\therefore$ Strain produced in the filament $= \dfrac{\delta ab}{ab}$

$= \dfrac{a'b' - ab}{ab}$

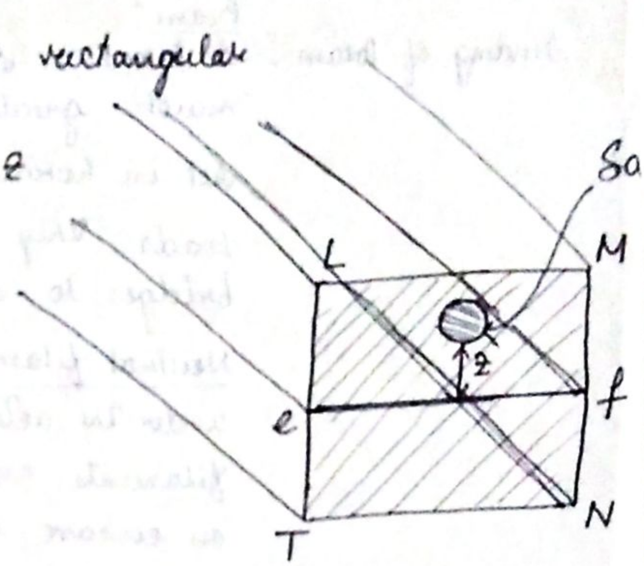
$= \dfrac{z\theta}{R\theta}$

$= \dfrac{z}{R}$

$\therefore$ strain produced $= \frac{z}{R}$ [$\because R =$ const for a particular beam]

$\boxed{\text{strain produced} \propto z}$ (distance of the filament from neutral axis).

Now, let us take into consideration the rectangular cross section of the beam, LMNT. A small area δa, at a distance z is considered at on LMNT. So strain produced in the filament passing through δa, is $\frac{z}{R}$.



$\therefore$ Young's modulus for the material of the beam $Y = \frac{\text{Normal Stress}}{\text{Longitudinal Strain}}$

$\Rightarrow Y = \frac{\text{stress}}{\left(\frac{z}{R}\right)}$

$\Rightarrow$ stress $= Y\frac{z}{R}$

$\Rightarrow$ Force $= \left(\frac{Yz}{R}\right)$ area $= \frac{Yz}{R}\delta a$

$\therefore$ Moment of this force about 'ef' $= \left(\frac{Yz}{R}\delta a\right) \cdot z$

$= \frac{Y}{R} z^2 \delta a \rightarrow (1)$

$\therefore$ Total moment of this force (1) for the whole beam (ie for cross section LMNT) is $= \sum \frac{Y}{R} \delta a z^2 = \frac{Y}{R} \sum z^2 \delta a$

This is nothing, but the bending moment of the beam.

$\therefore$ Bending moment (B.M) $= \frac{Y}{R}\sum z^2 \delta a$

$\boxed{B.M = \frac{Y}{R} I_g}$

$\boxed{B.M = \frac{Y}{R} aK^2}$

[Here $I_g = \sum z^2 \delta a = aK^2$ is called geometrical moment of inertia of the beam,

a = area of LMNT

K = Radius of gyration

Here $\boxed{YI_g = YaK^2}$ = flexural rigidity for the beam.

$\therefore$ Bending Moment $= \frac{\text{Flexural Rigidity}}{R}$.

Case - I For a beam of rectangular cross section

area $= b \times d$

$\therefore I_g = ak^2 = (b \times d) \times \left(\frac{d^2}{12}\right)$

$= \frac{bd^3}{12}$

$\therefore \boxed{B.M = \frac{Ybd^3}{12R}}$ ← For rectangle

Case - II For a beam of circular cross section of radius r.

area $a = \pi r^2$

$K = \frac{r^2}{4}$.

$\therefore I_g = \pi \frac{r^4}{4}$ $\quad \therefore \boxed{B.M = \frac{Y\pi r^4}{4R}}$

→ For circle