Matrix : Solution of simultaneous equation

Let us consider ~~the~~ a system of n linear algebraic equations in n unknowns.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n = b_3$$
$$\vdots \qquad \vdots \qquad \vdots \qquad\qquad \vdots \qquad \vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n = b_n$$

where a_{ij} are known coefficients.

b_i are known right hand side values.

x_i are the unknowns to be calculated.

The matrix notation for this system of equations can be written as follows

$$AX = b$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

The matrix $[A|b]$ obtained by appending the column b to the matrix A, is called the augmented matrix

$$[A|b] = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & & & \vdots & \vdots \\ a_{n1} & \cdots & & a_{nn} & b_n \end{array}\right]$$

Q. Solve the system of equations

$$x_1 + 10x_2 - x_3 = 3$$
$$2x_1 + 3x_2 + 20x_3 = 7$$
$$10x_1 - x_2 + 2x_3 = 4$$

using Gauss Elimination method with partial pivoting.

Solⁿ The augmented matrix for the system is

$$\left[\begin{array}{ccc|c} 1 & 10 & -1 & 3 \\ 2 & 3 & 20 & 7 \\ 10 & -1 & 2 & 4 \end{array}\right]$$

$$R_1 \leftrightarrow R_3 \quad \left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 2 & 3 & 20 & 7 \\ 1 & 10 & -1 & 3 \end{array}\right]$$

$$R_3 - R_1/10 \quad \left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 2 & 3 & 20 & 7 \\ 0 & (10+\frac{1}{10}) & (-1-\frac{2}{10}) & (3-\frac{2}{5}) \end{array}\right]$$

$$R_2 - \frac{R_1}{5} \quad \left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 2 & 3 & 20 & 7 \\ 0 & 1.1 & -1.2 & 2.6 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 0 & (3+\frac{1}{5}) & (20-\frac{2}{5}) & (7-\frac{4}{5}) \\ 0 & 1.1 & -1.2 & 2.6 \end{array}\right]$$

$\frac{1.1}{1.1} \times + (-1) = 0$

$0 + 10$

$$\left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 0 & 3.2 & 19.6 & 6.2 \\ 0 & 1.1 & -1.2 & 2.6 \end{array}\right]$$

$$R_2 \leftrightarrow R_3 \left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 0 & 10.1 & -1.2 & 2.6 \\ 0 & 3.2 & 19.6 & 6.2 \end{array}\right]$$

$$R_3 - \left(\frac{3.2}{10.1}\right) R_2 : \left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 0 & 10.1 & -1.2 & 2.6 \\ 0 & 0 & 19.98020 & 5.37624 \end{array}\right]$$

$$\therefore \quad x_3 = \frac{b_{3}}{a_{33}} = \frac{5.37624}{19.98020}$$

$$= 0.26908$$

$$x_2 = \left(b_2^{(1)} - a_{23}^{(1)} x_3\right) / a_{22}^{(1)} = 0.28940$$

$$x_3 = \left(b_1 - a_{12} x_2 - a_{13} x_3\right) / a_{11} = 0.37512$$

$$= \frac{1}{10}\left(4 + 0.2894 - 2(0.26908)\right)$$