Q. Solve the following system of equations using Gauss - Jordan method (i) without pivoting (ii) with partial pivoting

$$x_1 + x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

Keerthana

9.5 / 15

Sol^n

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$$

∴ Augmented matrix $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 4 & 3 & -1 & 6 \\ 3 & 5 & 3 & 4 \end{array}\right]$

(i) Without pivoting

$R_2 \to R_2 - 4R_1$: $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & +2 \\ 3 & 5 & 3 & 4 \end{array}\right]$

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$R_3 \to R_3 - 3R_1$; $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & +1 & +5 & \mp 2 \\ 0 & 2 & 0 & 1 \end{array}\right]$

$R_3 \to R_1 + R_3$: $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 5 & -2 \\ 1 & 3 & 1 & 2 \end{array}\right]$

$R_1 \to R_1 - R_2$ $\left[\begin{array}{ccc|c} 1 & 0 & -4 & 3 \\ 0 & 1 & 5 & -2 \\ 1 & 3 & 1 & 2 \end{array}\right]$

$R_3 \to R_3 - 3R_2$ $\left[\begin{array}{ccc|c} 1 & 0 & -4 & 3 \\ 0 & 1 & 5 & -2 \\ 1 & 0 & -14 & 8 \end{array}\right]$

$R_3 \to R_3 - R_1$ $\left[\begin{array}{ccc|c} 1 & 0 & -4 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & -10 & 5 \end{array}\right]$

$$R_3 \to R_3/(-10): \left[\begin{array}{ccc|c} 1 & 0 & -4 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right]$$

$-\frac{1}{2} - 5(-2)$

$= -\frac{1}{2} + 10$

$= \frac{20-1}{2} = \frac{19}{2}$

$3 + \cancel{4} \cdot \frac{19}{\cancel{2}}$

$3 + 38$

$= 41$

$$R_2 \to R_2 - 5R_3: \left[\begin{array}{ccc|c} 1 & 0 & -4 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & \frac{19}{2} \end{array}\right]$$

$$R_1 \to R_1 + 4R_3 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 41 \end{array}\right]$$

$x_1 = 3$

$x_2 = -2$

$x_3 = 41$

$$R_2 \to R_2 - 4R_1, \quad R_3 \to R_3 - 3R_1 \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & 2 \\ 0 & 2 & 0 & 1 \end{array}\right]$$

$$R_1 \to R_1 + R_2, \quad R_3 \to R_3 + 2R_2 \quad \left[\begin{array}{ccc|c} 1 & 0 & -4 & 3 \\ 0 & -1 & -5 & 2 \\ 0 & 0 & -10 & 5 \end{array}\right]$$

$$R_1 - \left(\frac{4}{10}\right)R_3, \quad R_2 - \left(\frac{5}{10}\right)R_3 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -\frac{1}{2} \\ 0 & 0 & -10 & 5 \end{array}\right]$$

$$R_2 \to -R_2, \quad R_3 \to \left(\frac{R_3}{-10}\right) \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right]$$

$\therefore$

$x_1 = d_1 = 1$

$x_2 = d_2 = \frac{1}{2}$

$x_3 = d_3 = -\frac{1}{2}$

By partial pivoting

$$R_1 \leftrightarrow R_2: \left[\begin{array}{ccc|c} 4 & 3 & -1 & 6 \\ 1 & 1 & 1 & 1 \\ 3 & 5 & 3 & 4 \end{array}\right]$$

max

$$R_1 \to R_1/4 \quad \left[\begin{array}{ccc|c} 1 & 3/4 & -1/4 & \frac{3}{2} \\ 1 & 1 & 1 & 1 \\ 3 & 5 & 3 & 4 \end{array}\right]$$

$$R_2 \to R_2 - R_1 \quad \left[\begin{array}{ccc|c} 1 & 3/4 & -1/4 & 3/2 \\ 0 & 1/4 & 5/4 & -1/2 \\ 3 & 5 & 3 & 4 \end{array}\right]$$

$$R_3 \to R_3 - 3R_1: \left[\begin{array}{ccc|c} 1 & 3/4 & -1/4 & 3/2 \\ 0 & \boxed{1/4} & 5/4 & -1/2 \\ 0 & \boxed{11/4} & 15/4 & -1/2 \end{array}\right]$$

$$R_3 \leftrightarrow R_2 \quad \left[\begin{array}{ccc|c} 1 & 3/4 & -1/4 & 3/2 \\ 0 & 11/4 & 15/4 & -1/2 \\ 0 & 1/4 & 5/4 & -1/2 \end{array}\right]$$

$$R_2 \to R_2 \frac{4}{11} \quad \left[\begin{array}{ccc|c} 1 & 3/4 & -1/4 & 3/2 \\ 0 & 1 & 15/11 & -2/11 \\ 0 & 1/4 & 5/4 & -1/2 \end{array}\right]$$

$$R_1 \to R_1 - \frac{3}{4}R_2 \quad \left[\begin{array}{ccc|c} 1 & 0 & -14/11 & 18/11 \\ 0 & 1 & 15/11 & -2/11 \\ 0 & 1/4 & 5/4 & -1/2 \end{array}\right]$$

$$R_3 \to R_3 - \frac{1}{4}R_2: \left[\begin{array}{ccc|c} 1 & 0 & -14/11 & 18/11 \\ 0 & 1 & 15/11 & -2/11 \\ 0 & 0 & 10/11 & -5/11 \end{array}\right]$$

$$R_3 \to R_3 \frac{11}{10} \quad \left[\begin{array}{ccc|c} 1 & 0 & -14/11 & 18/11 \\ 0 & 1 & 15/11 & -2/11 \\ 0 & 0 & 1 & -1/2 \end{array}\right]$$

$-\frac{5}{11} \times \frac{11}{10} = \frac{1}{2}$

$5 - \frac{9}{4}$

$3 + \frac{3}{4} = \frac{15}{4}$

$\frac{18}{11} + \left(-\frac{1}{2}\right)\frac{14}{11} = \frac{18}{11} - \frac{14}{22} = \frac{36-14}{22} = \frac{22}{22}$

$4 - 3\frac{3}{2} = 4 - \frac{9}{2} = \frac{-1}{2}$

Done (first)

$-\frac{1}{4} - \frac{3}{4}\frac{15}{11} = \frac{-11-45}{44} = \frac{-56}{44} = \frac{-14}{11}$

$\frac{3}{2} + \frac{3}{4}\left(\frac{2}{11}\right) = \frac{3}{2} + \frac{6}{44} = \frac{66+6}{44} = \frac{72}{44} = \frac{18}{11}$

$\frac{5}{4} - \frac{15}{44} = \frac{55-15}{44} = \frac{40}{44} = \frac{10}{11}$

$-\frac{1}{2} + \frac{2}{44} = \frac{-22+2}{44} = \frac{-20}{44} = \frac{-5}{11}$

$R_1 \to R_1 + \frac{14}{11} R_3$: $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 15/11 & -2/11 \\ 0 & 0 & 1 & -1/2 \end{array}\right]$

$R_2 \to R_2 - \frac{15}{11} R_3$: $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & +1/2 \\ 0 & 0 & 1 & -1/2 \end{array}\right]$

$-\frac{2}{11} + \frac{15}{11} \times \left(\frac{1}{2}\right)$

$= \frac{-4+15}{22}$

$= \frac{11}{22}$

$= 1/2$

$\therefore \left.\begin{array}{l} x_1 = 1 \\ x_2 = 1/2 \\ x_3 = -1/2 \end{array}\right\}$ Solutions