Interpolation with evenly spaced points :

• Newton's Forward Difference interpolation formula :

Let us consider a given data with a step length h.

Now, the Newton's forward difference interpolation formula is

$$f(x) = f(x_0) + (x-x_0)\frac{\Delta f(x_0)}{\lfloor 1\, h} + (x-x_0)(x-x_1)\frac{\Delta^2 f(x_0)}{\lfloor 2\, h^2} +$$

$$\cdots + \frac{(x-x_0)(x-x_1)\cdots\cdots(x-x_{n-1})}{\lfloor n-1}$$

Here $x_i = (x_0 + ih)$

Putting and $x = x_0 + sh$

$$f(x) = f(x_0+sh)$$

$$= f(x_0) + s\Delta f_0 + \frac{s(s-1)}{\lfloor 2}\Delta^2 f_0 + \frac{s(s-1)(s-2)\Delta^3 f_0}{\lfloor 3}$$

$$+ \cdots + \frac{s(s-1)\cdots\cdots(s-n+1)}{\lfloor n}\Delta^n f_0$$

$$= \left({}^sC_0 f(x_0) + {}^sC_1 \Delta f_0 + {}^sC_2 \Delta^2 f_0 + \cdots + {}^sC_n \Delta^n f_0\right)$$

Here $s = \left[\left(\frac{x-x_0}{h}\right)\right] > 0$

Error in Interpolation

$$E_n(f,x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{\lfloor n+1} f^{n+1}(\xi)$$

$$\Rightarrow E(f,x) = \frac{s(s-1)(s-2)\cdots(s-n)}{\lfloor n+1} h^{n+1} f^{n+1}(\xi)$$

$$\Rightarrow \boxed{E(f,x) = s C_{n+1} h^{n+1} f^{n+1}(\xi)} \; ; \; 0 < \xi < n$$

Q. Using the following data construct the forward difference formula. Hence ~~evaluate~~ find $f(0.5)$.

x	-2	-1	0	1	2	3
$f(x)$	15	5	1	3	11	25

$$f(x) = f(x_0) + \frac{(x-x_0)}{\lfloor 1 h}\Delta f(x_0) + \frac{(x-x_0)(x-x_1)}{\lfloor 2 h^2}\Delta^2 f(x_0) + \cdots$$

$$= 15 + \frac{(x+2)}{(+1)}(-10) + (x+2)(x+1)\left(\frac{6}{2}\right)$$

$$= 15 - 10x - 20 + 3x^2 + 9x + 6$$

$$\Rightarrow f(x) = 3x^2 - x + 1$$

$$\therefore f(0.5) = 3\times(0.5)^2 - 0.5 + 1$$

$$\boxed{f(0.5) = 1.25}$$